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Sound generation in the ocean by breaking surface waves

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Various flow processes resulting from the breaking of ocean surface waves are examined in order to determine their relative efficiencies as sources of sound. Momentum fluctuation arising from splashing water sprays is identified as the major contributor to the underwater sound. It is shown that the splashing is more efficient in radiating sound than other processes, such as unsteady foaming that entrains air bubbles into water, and turbulent motions in the surface layer associated with compressibility of the entrained bubbles. A model is presented to estimate the sound power radiated in terms of parameters of the wind and surface wave field. Comparison of theory with measurements is made and good agreement is discovered.

1. Introduction

This paper examines the problem of sound generation in the ocean by fluid motions adjacent to the air–water interface, with particular reference to those resulting from the breaking of surface waves. Because of the complicated conditions in the real ocean it has long been suggested that a wide range of physical mechanisms are jointly responsible for the oceanic noise (see, e.g. Knudsen, Alford & Emling 1948; Wenz 1962). But it has also long been recognized that the underwater sound is somehow related to the violent motion of the ocean surface. In this paper we show that this relation is through the wave-breaking process; more specifically, through the splashing of water sprays in the breaking. As wind blows over the ocean surface, waves may break; water sprays are detached from surface waves into the turbulent airflow, and later fall down on to the water surface again to form patches of foam and entrain air bubbles into water. All these processes, as well as various motions of the entrained bubbles, can generate sound in one way or another. We will show, however, that it is the splashing of water sprays, which have a much heavier mean density than their surroundings and hence cause a rapid variation of momentum in the source flow, that is the major contributor to noise in deep water.

This sound generation by momentum variation is actually within the category of sound production by mixed turbulent flow in aeroacoustics. Ffowcs Williams (1986) has shown that, whenever mixed turbulent motions occur near a density interface, the dominant sound arises from momentum fluctuation due to the rapid density variation in the source flow. This principle can be used to interpret the generation process of underwater sound in the presence of waves breaking on the ocean surface. Since there are some differences between our ocean sound problem and that considered by Ffowcs Williams – the change in sound speed across the density interface and the entrainment of air bubbles, for example – we will derive the theory in a different way to take account of all these effects. Our theory is expressed in such a way that the oceanic sound is identified as being generated by two distinct mechanisms: the unsteady momentum flux at the ocean surface, and the effects of bubble-induced compressibility of the source flow.

In fact, our result for the momentum-induced sound is essentially the Ffowcs Williams theory, except for a slightly different proportionality constant that completely accounts for the effect of change in sound speed across the air–water interface and is precisely equal to the transmission coefficient of a plane interface for sound waves propagating from air into water. This sound, of dipole type in general, is shown to be the dominant sound produced by breaking waves. We will demonstrate that the breaking of waves is crucially important and that the dipole sources degenerate to relatively ineffective quadrupoles when there is no wave breaking.

Bubbles in a turbulent flow are known to generate sound by volumetrically responding to the fluctuating turbulent pressure and by inducing momentum defects in the source flow. Both these mechanisms are very efficient acoustic sources in certain situations and one would naturally expect them to feature in the surface-wave sound-generation problem. But to our surprise our studies have led us to the view that bubble motions are actually irrelevant to this problem. We will show that their sound is essentially of smaller order than that from water sprays. This is because the bubbles are all located near a density interface that is almost pressure-release; the monopoles interfere destructively with their images.

Having identified splashing water sprays as the dominant acoustic source, we finally apply the theory to predict the sound power from this mechanism in a model of the real ocean. Owing to the recent progress in understanding the relation between statistical properties of breaking waves and winds (Phillips 1985), we are able to estimate the sound power radiated by splashing sprays in terms of the wind speed and the minimum phase speed of waves capable of producing sprays. We predict a quadratic dependence of the overall sound power on wind speed, which is well consistent with observations in the natural ocean. The extensive existing measurements of underwater sound suggest that the sound-pressure spectra have different wind-speed dependencies at different frequencies (e.g. Perrone 1969; Morris 1978). For decreasing frequencies, a decreasing degree of dependence of deep ocean noise spectra on wind has been observed. In general the relation can be described as a power law with index scattered in the range 1.5–3.6. But when the pressure spectra are used to evaluate the overall sound power, the power output is found to follow our quadratic-dependence theory very well. Our model predicts that in a storm of typical wind 15 m/s, the peak frequency of the surface-generated sound is about 500 Hz, a value consistent with observation in the ocean (Wenz 1962), though a quantitative detailed verification is almost impossible because the measured sound cannot be distinguished as coming exclusively from sprays.

2. The development of the theory

We consider a configuration in which the air and water are separated by an infinite interface. Sound is generated by sources in the proximity of this interface, such as turbulent motions of the bubbly flow and the splashing water sprays from breaking waves. Far from the source layer the fluids are at rest apart from sound waves from the sources. We choose two horizontal control surfaces $x_3 = 0$ and $x_3 = \Delta$, x_3 being the vertical coordinate with an upward positive direction, to separate the mixed turbulence sources from the only acoustically disturbed water and air. Sources are then all confined to the region $0 \leqslant x_3 \leqslant \Delta$ (see figure 1). The motion in the region

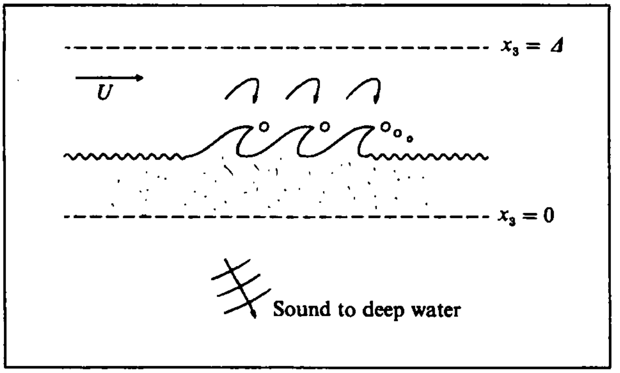

FIGURE 1. The geometry of the problem. Sound is generated by flow processes on the sea surface.

below $x_3 = 0$ can therefore be specified by the ordinary homogeneous wave equation in terms of sound-pressure fluctuation $p(\boldsymbol{x}, t)$, namely,

$$\frac{\partial^2 p(\boldsymbol{x}, t)}{\partial t^2} - c_w^2 \nabla^2 p(\boldsymbol{x}, t) = 0, \tag{2.1}$$

in $x_3 \leqslant 0$, c_w denoting the constant sound speed in water. We take Fourier transformations in the horizontal coordinates x_α and time t so that (2.1) implies that

$$\frac{\partial^2 \hat{p}(x_3)}{\partial x_3^2} + \gamma_w^2 \hat{p}(x_3) = 0, \tag{2.2}$$

where $\gamma_w^2 = \omega^2/c_w^2 - k_\alpha^2$ and the symbol $\hat{}$ means that the quantity indicated is in the wavenumber–frequency space. This equation can be solved directly. The result can be expressed in terms of pressure fluctuations on the control surface $x_3 = 0$. Taking account of the radiation condition at $x_3 \to -\infty$, we must have as the solution of (2.2)

$$\hat{p}(x_3) = \hat{p}(0)\, \mathrm{e}^{-\mathrm{i}\gamma_w x_3}, \tag{2.3}$$

with the branches of γ_w chosen such that when real they have the same sign as ω and then purely imaginary $\mathrm{Im}\,(\gamma_w)$ is always positive. This choice guarantees that sound waves are outgoing and finite at $x_3 \to -\infty$. From the solution (2.3) and by making use of the vertical component of the linearized momentum equation $\rho_w \partial u_3/\partial t + \partial p/\partial x_3 = 0$, ρ_w being the mean density of water and u_3 the vertical velocity, it is easy to show that

$$\hat{p}(0) = \frac{-\omega \rho_w}{\gamma_w} \hat{u}_3(0), \tag{2.4}$$

where $\hat{u}_3$ denotes the Fourier transform of u_3. Thus (2.3) can be alternatively written as

$$\hat{p}(x_3) = \frac{-\omega \rho_w}{\gamma_w} \hat{u}_3(0)\, \mathrm{e}^{-\mathrm{i}\gamma_w x_3}. \tag{2.5}$$

The foregoing procedure is equally applicable to the region $x_3 \geqslant \Delta$ where the only motion consists of sound waves which are governed by (2.1) with c_w replaced by c_a, the constant atmospheric sound speed. Corresponding to (2.4) we find that on $x_3 = \Delta$

$$\hat{p}(\Delta) = \frac{\omega \rho_a}{\gamma_a} \hat{u}_3(\Delta), \tag{2.6}$$

where ρ_a is the mean aerial density and $\gamma_a^2 = \omega^2/c_a^2 - k_\alpha^2$, the branches of which are chosen similarly to those of γ_w to ensure the radiation condition at $x_3 \to +\infty$. By rearranging (2.6), (2.5) and (2.3), and appropriately combining them, it can be deduced that

$$\omega(\hat{u}_3(\Delta) - \hat{u}_3(0)) = \frac{\gamma_a}{\rho_a}\hat{p}(\Delta) + \frac{\gamma_w}{\rho_w}\hat{p}(x_3)\,e^{i\gamma_w x_3},$$

and

$$\frac{\gamma_a}{\rho_a}(\hat{p}(\Delta) - \hat{p}(0)) = \omega\hat{u}_3(\Delta) - \frac{\gamma_a}{\rho_a}\hat{p}(x_3)\,e^{i\gamma_w x_3}.$$

The subtraction of these two relations immediately yields

$$\hat{p}(x_3) = \frac{\rho_w\gamma_a}{\rho_w\gamma_a + \rho_a\gamma_w}\left(\frac{\omega\rho_a}{\gamma_a}[\hat{u}_3(\Delta) - \hat{u}_3(0)] - [\hat{p}(\Delta) - \hat{p}(0)]\right)e^{-i\gamma_w x_3}. \tag{2.7}$$

Since

$$\hat{p}(\Delta) - \hat{p}(0) = \int_0^\Delta \frac{\partial\hat{p}(y_3)}{\partial y_3}\,dy_3,$$

and

$$\hat{u}_3(\Delta) - \hat{u}_3(0) = \int_0^\Delta \frac{\partial\hat{u}_3(y_3)}{\partial y_3}\,dy_3,$$

(2.7) is equivalent to

$$\hat{p}(x_3) = \int_0^\Delta \frac{\rho_w\gamma_a}{\rho_w\gamma_a + \rho_a\gamma_w}\left(\frac{\omega\rho_a}{\gamma_a}\frac{\partial\hat{u}_3(y_3)}{\partial y_3} - \frac{\partial\hat{p}(y_3)}{\partial y_3}\right)e^{-i\gamma_w x_3}\,dy_3.$$

Now the inverse Fourier transformation can be used to find the sound pressure in the region $x_3 \leqslant 0$, that is,

$$p(\boldsymbol{x}, t) = \frac{1}{(2\pi)^3}\int_0^\Delta \int_\infty \frac{\rho_w\gamma_a}{\rho_w\gamma_a + \rho_a\gamma_w}\left(\frac{\omega\rho_a}{\gamma_a}\frac{\partial u_3(\boldsymbol{y}, \tau)}{\partial y_3} - \frac{\partial p(\boldsymbol{y}, \tau)}{\partial y_3}\right)$$
$$\times\, e^{i[k_\alpha(y_\alpha - x_\alpha) + \omega(\tau - t) - \gamma_w x_3]}\,d^3\boldsymbol{y}\,d^2k_\alpha\,d\tau\,d\omega, \tag{2.8}$$

where $\hat{p}$ and $\hat{u}_3$ have also been expressed in terms of their Fourier transforms in the physical space.

The $\boldsymbol{y}$-integral in (2.8) is to be performed over the source region $0 \leqslant y_3 \leqslant \Delta$, in which the motions of fluids are highly nonlinear, so that the complete set of equations of fluid motions must be used. These are, if viscosity and heat conductivity are ignored,

$$\frac{\partial u_3}{\partial y_3} + \frac{\partial u_\alpha}{\partial y_\alpha} + \frac{1}{\rho}\frac{D\rho}{D\tau} = 0, \quad \frac{\partial p}{\partial y_3} + \rho\frac{Du_3}{D\tau} = 0, \quad \frac{D\rho}{D\tau} = \frac{1}{c_m^2}\frac{Dp}{D\tau}, \tag{2.9}$$

where c_m is the sound speed in the turbulence mixture, which may differ from both c_a and c_w because of the presence of air bubbles. From these equations we derive the identity

$$\frac{\omega\rho_a}{\gamma_a}\frac{\partial u_3}{\partial y_3} - \frac{\partial p}{\partial y_3} = \rho\frac{Du_3}{D\tau} - \frac{\omega\rho_a}{\gamma_a}\left(\frac{\partial u_\alpha}{\partial y_\alpha} + \frac{1}{\rho c_m^2}\frac{Dp}{D\tau}\right),$$

so that (2.8) becomes

$$p(\boldsymbol{x}, t) = \frac{1}{(2\pi)^3}\int_0^\Delta \int_\infty \frac{\rho_w\gamma_a}{\rho_w\gamma_a + \rho_a\gamma_w}\left(\rho\frac{Du_3}{D\tau} - \frac{\omega\rho_a}{\gamma_a}\left[\frac{\partial u_\alpha}{\partial y_\alpha} + \frac{1}{\rho c_m^2}\frac{Dp}{D\tau}\right]\right)$$
$$\times\, e^{i[k_\alpha(y_\alpha - x_\alpha) + \omega(\tau - t) - \gamma_w x_3]}\,d^3\boldsymbol{y}\,d^2k_\alpha\,d\tau\,d\omega.$$

Since we are mainly concerned with sound in the deep water, the method of two-dimensional stationary phase (Jones 1982) can be applied directly to the k_α-integration. The result can then be simplified still further by carrying out the ω-integral with a result containing a δ-function and then utilizing it to evaluate the integration with respect to τ. This leads to

$$p(\boldsymbol{x},t) = \frac{-\cos\theta T(\theta)}{2\pi c_{\mathrm{w}}|\boldsymbol{x}|}\frac{\partial}{\partial t}\int_{V_0}\left[\rho\frac{\mathrm{D}u_3}{\mathrm{D}\tau} - \frac{\rho_{\mathrm{a}} c_{\mathrm{a}}}{(1-\sin^2\theta c_{\mathrm{a}}^2/c_{\mathrm{w}}^2)^{\frac{1}{2}}}\left(\frac{\partial u_\alpha}{\partial y_\alpha} + \frac{1}{\rho c_{\mathrm{m}}^2}\frac{\mathrm{D}p}{\mathrm{D}\tau}\right)\right]\mathrm{d}^3\boldsymbol{y}, \quad (2.10)$$

where $\cos\theta = x_3/|\boldsymbol{x}|$ and we have denoted by V_0 the turbulence source region. The bracket [ ] has what is by now the conventional retarded-time implication. $T(\theta)$ represents the transmission coefficient of a plane air–water interface for sound waves propagating from air into water, and is defined by

$$T(\theta) = \left(1-\sin^2\theta\frac{c_{\mathrm{a}}^2}{c_{\mathrm{w}}^2}\right)^{\frac{1}{2}} \Big/ \left[\left(1-\sin^2\theta\frac{c_{\mathrm{a}}^2}{c_{\mathrm{w}}^2}\right)^{\frac{1}{2}} - \cos\theta\frac{\rho_{\mathrm{a}} c_{\mathrm{a}}}{\rho_{\mathrm{w}} c_{\mathrm{w}}}\right].$$

The solution (2.10), derived from the complete set of equations of fluid motion, is an exact representation of the far-field sound. It can be simplified once it is recognized that the term containing $\partial u_\alpha/\partial y_\alpha$ in the integrand is actually negligible. To demonstrate this, we derive the relation, through the use of the retarded-time implication and the set of equations (2.9),

$$\left[\frac{\partial u_\alpha}{\partial y_\alpha}\right] = \left[\frac{-x_\alpha}{c_{\mathrm{w}}|\boldsymbol{x}|}\frac{\mathrm{D}u_\alpha}{\mathrm{D}\tau}\right] + S, \quad (2.11)$$

where

$$S = \frac{\partial[u_\alpha]}{\partial y_\alpha} + \frac{x_\alpha}{c_{\mathrm{w}}|\boldsymbol{x}|}\frac{\partial[u_\alpha u_i]}{\partial y_i} - \frac{x_\alpha x_\beta}{c_{\mathrm{w}}^2|\boldsymbol{x}|^2}\left[\frac{\partial u_\alpha u_\beta}{\partial \tau}\right] - \frac{x_\alpha}{c_{\mathrm{w}}|\boldsymbol{x}|}\left[\frac{u_\alpha}{c_{\mathrm{m}}^2\rho}\frac{\mathrm{D}p}{\mathrm{D}\tau}\right]$$

is negligible. The first two terms are in divergence form so that they both integrate to zero because the bounding surfaces of V_0 are linear and u_α vanishes at $|y_\alpha| \to \infty$. The third term is evidently a quadrupole field; it is smaller than the leading term in (2.11) by $M = u/c_{\mathrm{w}} \ll 1$, u being the typical turbulence velocity. In considering the last term, we scale the pressure perturbation in the source region as $\rho_{\mathrm{w}} uU$, U denoting the typical mean flow velocity, the uniform wind speed for example. Thus the ratio of this term to the leading term in (2.11) is of the order uU/c_{m}^2. Now c_{m} may be much lower than both c_{a} and c_{w} (owing to the presence of air bubbles in the flow) and may be of the same order as U, but it can never drop below u. In fact, c_{m} is at least one order of magnitude bigger than u; uU/c_{m}^2 is then much smaller than one. This completes the proof that S is negligible. When (2.11) is substituted into (2.10), it becomes clear that the term containing $\partial u_\alpha/\partial y_\alpha$ is exceedingly small. Ffowcs Williams (1986) has given a general proof that it is the divergence property that makes its integrated effect vanish. In fact, it is even more straightforward in our ocean-sound problem to show this by a direct comparison of the integrands. Considering (2.11), it follows that the first term and the term proportional to $\partial u_\alpha/\partial y_\alpha$ in (2.10) can respectively be scaled as

$$\rho\frac{\mathrm{D}u_3}{\mathrm{D}\tau} \quad \text{and} \quad \frac{\rho_{\mathrm{a}} c_{\mathrm{a}}}{c_{\mathrm{w}}}\frac{\mathrm{D}u_\alpha}{\mathrm{D}\tau}.$$

On the ocean surface it is the momentum excess of flow elements with distinctly different mean density from their surroundings that generates sound (as will be seen in the following sections). Therefore the density ρ should be scaled on ρ_{w}, the mean water density, and hence the first term is obviously of the order $\rho_{\mathrm{w}} c_{\mathrm{w}}/\rho_{\mathrm{a}} c_{\mathrm{a}} \approx 10^4$

larger than the second. Thus the term proportional to $\partial u_\alpha/\partial y_\alpha$ can be neglected and we eventually have

$$p(\boldsymbol{x},t) = \frac{-\cos\theta T(\theta)}{2\pi c_{\mathrm{w}}|\boldsymbol{x}|}\frac{\partial}{\partial t}\int_{V_0}\left[\rho_0\frac{\mathrm{D}u_3}{\mathrm{D}\tau}+\rho'\frac{\mathrm{D}u_3}{\mathrm{D}\tau}+\frac{\rho_{\mathrm{a}}c_{\mathrm{a}}/\rho c_{\mathrm{m}}^2}{(1-\sin^2\theta c_{\mathrm{a}}^2/c_{\mathrm{w}}^2)^{\frac{1}{2}}}\frac{\mathrm{D}p}{\mathrm{D}\tau}\right]\mathrm{d}^3\boldsymbol{y}, \quad (2.12)$$

where we have split the density of a fluid particle at $\boldsymbol{x}$ into its mean value $\rho_0(\boldsymbol{x})$, plus the small deviation $\rho'(\boldsymbol{x},t)$ due to compressibility; $\rho(\boldsymbol{x},t) = \rho_0(\boldsymbol{x})+\rho'(\boldsymbol{x},t)$.

This solution is consistent with the Ffowcs Williams (1986) theory. In his problem of aeroacoustics, bubbles are not considered, so that c_{m} in (2.12) should be replaced by either c_{a} or c_{w}. The term inversely proportional to c_{m}^2 is then clearly much smaller than the first term, smaller at least by the factor $(\rho_{\mathrm{a}}/\rho_{\mathrm{w}})(U/c_{\mathrm{a}})$. If we further let $c_{\mathrm{a}} = c_{\mathrm{w}}$, Ffowcs Williams' equation (4.24) is precisely recovered. The change caused by allowing the difference in sound speed across the air–water interface is all confined to the transmission coefficient $T(\theta)$. Since bubbles are commonly observed in the ocean surface layer (Wenz 1962), it is obviously of interest to see whether they can produce an appreciable sound field. Hence we have retained the term inversely proportional to c_{m}^2.

Now, the two source mechanisms are clearly revealed. The first is similar to that discussed by Ffowcs Williams in the sound production by mixed jet flow, that is, the unsteady momentum fluctuation in the source flow. The sound of this source, given by the first term of (2.12), is in general of dipole type, the axes of the dipole sources being perpendicular to the mean position of the air–water interface, and bigger in magnitude than that from the non-mixed turbulence. The second kind of source is accounted for by the second and last terms, and is associated with compressible motions in the source region. In the ocean-sound problem, compressible motions in the surface layer mainly result from air bubbles, their volumetric response to turbulent pressure fluctuation for example. Thus this mechanism may be important only in the bubbly mixture, where the fluids can possibly be compressed in such a way as to significantly influence the sound production.

3. The dominant source of sound during the breaking

When surface waves are breaking, clouds of water sprays are thrown up into the airflow. The splashing water sprays are highly sporadic and sparse. Within a spray the mean density is very high, scaling on the mean water density ρ_{w}, while outside it is ρ_{a}. The density and hence the momentum distribution in the source region is then sharply discontinuous in space and time. In this case, we identify the sound from unsteady momentum fluctuation as the dominant sound. This becomes evident if we compare the relative magnitude of the three terms in (2.12). On scaling p as $\rho_{\mathrm{w}}uU$ and therefore ρ' as $\rho_{\mathrm{w}}uU/c_{\mathrm{m}}^2$ by definition, the ratios of the first to the second and the third terms are, respectively,

$$\frac{c_{\mathrm{m}}^2}{uU} \quad \text{and} \quad \frac{\rho_{\mathrm{w}}}{\rho_{\mathrm{a}}}\left(\frac{c_{\mathrm{m}}}{c_{\mathrm{a}}}\right)^2\frac{c_{\mathrm{a}}}{U}. \quad (3.1)$$

The first ratio is very much bigger than one, because c_{m} is always much bigger than u even at its minimum value, which is of the same order as U, while the second is at least of the same order as the inverse of the wind Mach number U/c_{a}. Hence the dominance of the first term in (2.12), that is, sources connected with momentum flux on the ocean surface, is established; compressible motions of the bubbles cannot be

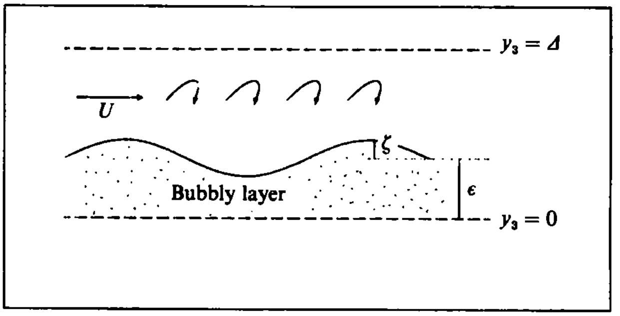

FIGURE 2. A continuous bubbly layer under the airflow.

important sources of sound. In fact, momentum defects due to the presence of bubble clouds in water are also negligible in comparison with that from splashing water sprays, so that the effects of the bubbles can be completely ignored as far as sound generation is concerned. We will show in the next section exactly why the bubbles are weak acoustic sources in our ocean-sound problem, but demonstrate first in what follows the crucial importance to the momentum-induced sound of the discontinuous variation in ρ_0, the density which the source flow would have in an exactly incompressible flow. We show that the momentum-induced sound degrades to higher order and smaller magnitude if there are no breaking waves on the ocean surface even when the surface is non-linearly disturbed.

To this end, we consider a bubbly turbulence layer of mean thickness ϵ and horizontally extensive and continuous. Hence ρ_0 varies abruptly only across the interfaces between the layer and the surrounding fluids, airflow above and pure water below. We express the bubble concentration β, the fraction of unit volume of the mixture which is occupied by the bubbles, as β_0 supplemented by a fluctuating part β' that accounts for motions of the bubbles associated with compressibility. Here β_0 is the concentration that would be caused by the bubbles if they were all rigid. β_0 is presumed homogeneous and constant (much less than one) inside the bubbly layer. Hence the interfaces between the layer and its surrounding fluids are modelled as discontinuities in β_0, β_0 being equal to one in the airflow, a small constant in the bubbly layer, and zero in the pure water. The density within the bubbly layer can be approximated by $\beta\rho_a + (1-\beta)\rho_w$ (the neglected terms being of the order c_m^2/c_a^2 smaller), or equivalently

$$\rho = \beta_0\rho_a + (1-\beta_0)\rho_w + (\rho_a - \rho_w)\beta', \tag{3.2}$$

from which we can deduce the density

$$\rho_0 = \beta_0\rho_a + (1-\beta_0)\rho_w + (\rho_a - \rho_w)(1-\beta_0)H(y_3 - \epsilon - \zeta) \tag{3.3}$$

in the source region V_0, where H denotes the Heaviside unit function and $y_3 = \epsilon + \zeta$ is the upper bounding surface of the bubbly layer. In deriving (3.3) we have taken the lower bounding surface of the layer as the linear control surface $y_3 = 0$ as shown in figure 2. This is legitimate because motions on this surface are very small in comparison with those on $y_3 = \epsilon + \zeta$, which models the ocean surface, due to the exponentially decaying property of ocean waves. The deformation ζ of the interface between the bubbly layer and the airflow should be in general determined by the combination of gravity waves and waveguide effects arising from multiple reflections of waves in the bubbly layer that has a sound speed much lower than those on either side of it. These guided waves, trapped in the layer, may be of high energy level, and propagate horizontally just like gravity waves. However, we find in the Appendix that the motion of the ocean surface due to waveguide effects is vanishingly small in comparison with that due to gravity waves. Hence we can simply regard ζ as the ocean-surface displacement due to gravity waves.

Since any fluid element on a continuous ocean surface will always remain on it, the material derivative of the Heaviside function in (3.3) vanishes, so that $D\rho_0/D\tau = 0$ and thus ρ_0 in the first term of the integrand of (2.12) can be taken inside the differential operator. The momentum-induced sound thus becomes

$$p(\boldsymbol{x},t) = \frac{-\cos\theta T(\theta)}{2\pi c_{\mathrm{w}}|\boldsymbol{x}|}\frac{\partial}{\partial t}\int_{V_0}\left[\frac{\partial\rho_0 u_3}{\partial t}+u_i\frac{\partial\rho_0 u_3}{\partial y_i}\right]\mathrm{d}^3\boldsymbol{y}. \tag{3.4}$$

The second term in the integrand can be further rewritten, by utilizing the retarded time implication and (2.9), as

$$\left[u_i\frac{\partial\rho_0 u_3}{\partial y_i}\right] = \frac{\partial[\rho_0 u_3 u_i]}{\partial y_i}-\frac{x_\alpha}{c_{\mathrm{w}}|\boldsymbol{x}|}\left[\frac{\partial\rho_0 u_3 u_\alpha}{\partial\tau}\right]+\left[\frac{\rho_0 u_3}{\rho c_{\mathrm{m}}^2}\frac{\mathrm{D}p}{\mathrm{D}\tau}\right].$$

These terms are all negligible. The first is a divergence; it integrates to zero because $u_3 u_i$ vanishes on the linear boundary surfaces $y_3 = 0$, $y_3 = \Delta$ and $|y_\alpha| \rightarrow \infty$. The second is a quadrupole field of the same order as those already omitted in §2, while the last can be easily seen to be not bigger than the second and third term of (2.12), which have been demonstrated to be of negligible importance. If we denote by Q all quadrupole terms, (3.4) is then equivalent to

$$p(\boldsymbol{x},t) = \frac{-\cos\theta T(\theta)}{2\pi c_{\mathrm{w}}|\boldsymbol{x}|}\frac{\partial^2}{\partial t^2}\int_{V_0}[\rho_0 u_3]\,\mathrm{d}^3\boldsymbol{y}+Q. \tag{3.5}$$

When (3.3) is inserted into this, the constant part of ρ_0 yields a sound field that is given by the integration of $[u_3]$ over a fixed volume bounded by the linear surfaces $y_3 = 0$ and $y_3 = \Delta$, and is equivalent to the sound that would be radiated by a turbulent flow of constant mean density $\beta_0\rho_{\mathrm{a}}+(1-\beta_0)\rho_{\mathrm{w}} \approx \rho_{\mathrm{w}}$ adjacent to a linear density interface $y_3 = \Delta$. This sound has been shown by Ffowcs Williams (1986) to be a small sound (in the sense that it is not bigger than Q). The sound corresponding to the last term of (3.3) would also be equal to such a quadrupole field if the ocean surface is only linearly deformed, namely, if ζ in (3.3) is linear and the Heaviside function can be replaced by $H(y_3-\epsilon)$. But here we take a further step to consider nonlinear effects of the ocean-surface motion. We suppose that ζ is small but finite in magnitude (being small to ensure the continuity of the air–water interface). Hence we have, on substituting (3.3) into (3.5),

$$\int_{V_0}[\rho_0 u_3]\,\mathrm{d}^3\boldsymbol{y} = (\rho_{\mathrm{a}}-\rho_{\mathrm{w}})(1-\beta_0)\int_{y_\alpha}\left[\left(\int_\epsilon^\Delta-\int_{\epsilon+\zeta}^\epsilon\right)u_3\,\mathrm{d}y_3\right]\mathrm{d}^2y_\alpha. \tag{3.6}$$

The first integral from ϵ to Δ is actually over a fixed volume so that it is of the same order as Q. The second one can be expanded in a power series of ζ. The convergence of this expansion is guaranteed by the small slope of the surface displacement. By truncating the expansion at the term proportional to ζ^2 and denoting u_3 by $\partial\zeta/\partial\tau$, we find that

$$p(\boldsymbol{x},t) = (1-\beta_0)\frac{\cos\theta T(\theta)}{2\pi c_{\mathrm{w}}|\boldsymbol{x}|}\frac{\rho_{\mathrm{a}}-\rho_{\mathrm{w}}}{2}\frac{\partial^3}{\partial t^3}\int_{y_\alpha}[\zeta^2]\,\mathrm{d}^2y_\alpha+Q.$$

It is easy to identify the first term on the right as equation (2.11) of a companion paper (Guo 1987) multiplied by $(1-\beta_0)$ that is effectively unity. In that paper we have shown that this sound is always negligible in comparison with the direct turbulence sound Q as long as the ocean surface remains intact. Acoustic dipole sources due to momentum flux degrade to higher-order quadrupoles when there are no breaking waves on the ocean surface. This result can be regarded as a generalization of the conclusion that a turbulent flow near a linear surface can only radiate quadrupole sound (Ffowcs Williams 1965, 1986). We have shown that this is also true in the ocean-sound problem even when the surface is non-linearly deformed.

4. Bubble sound

Bubbles can be very efficient acoustic sources sometimes; they can introduce monopoles by volumetrically responding to the fluctuating turbulence pressure, and dipoles by inducing a rapidly varying density and momentum distribution. But in our problem of sound generation by surface waves, these two are both irrelevant; neither of them can contribute any appreciable sound. We show this in detail in this section. Let us first consider the sound produced by compressible motions of the bubbles. From (3.2) we see that the fluctuating part of the density ρ associated with compressibility is

$$\rho' = (\rho_a - \rho_w)\,\beta'. \tag{4.1}$$

The sound due to compressible motions is given by the second and third terms of (2.12),

$$p(\boldsymbol{x},t) = \frac{-\cos\theta T(\theta)}{2\pi c_w|\boldsymbol{x}|}\frac{\partial}{\partial t}\int_{V_0}\left[(\rho_a-\rho_w)\,\beta'\frac{\mathrm{D}u_3}{\mathrm{D}\tau}+\frac{\rho_a c_a/\rho c_m^2}{(1-\sin^2\theta c_a^2/c_w^2)^{\frac{1}{2}}}\frac{\mathrm{D}p}{\mathrm{D}\tau}\right]\mathrm{d}^3\boldsymbol{y}. \tag{4.2}$$

To calculate this sound, we need to estimate the magnitude of β'. For a bubbly flow embedded in an infinite region of water of uniform mean density, Crighton & Ffowcs Williams (1969) have derived an expression for β' which, in the case of forced motions of typical frequency much lower than the bubble resonance frequency ω_0, scales as

$$\beta' \sim \frac{-3\beta_0}{(a\omega_0)^2}\frac{p}{\rho_w}, \tag{4.3}$$

where a is the mean radius of the bubbles. They have also given the detailed expression for ω_0. We will use this scaling law in our problem where the bubbly flow is near the ocean surface. It can be justified in two ways. The first is to examine their derivations. It can be noticed that their argument leading to (4.3) is actually independent of any boundaries in the flow field. Alternatively, it can be checked by the use of the simple relation $(a\omega_0)^2 = 3\beta_0 c_m^2$ for β_0 neither too small nor too close to unity (Crighton & Ffowcs Williams), which reduces (4.3) to $\beta' = -p/c_m^2\rho_w$. From (4.1) the density fluctuation is approximately $-\rho_w\beta'$. Hence the relation (4.3) is in fact equivalent to $p = c_m^2\rho'$, the definition of sound speed in the bubbly liquid which is certainly independent of any boundaries in the flow. Either way, we see that (4.3) is also valid in our problem.

The acoustic power output can be calculated from (4.2) by integrating the intensity over a large hemisphere. Suppose that the turbulence source region V_0 extends horizontally over an area of dimension L^2. The typical eddy size within the region is denoted by l. Thus $p \sim \rho_w uU$ and $\partial/\partial t \sim \mathrm{D}/\mathrm{D}\tau$ can be scaled as a typical frequency u/l. The acoustic power can then be estimated in terms of these parameters. A radiation efficiency η can be conveniently defined by dividing the power output by the rate of working of the fluctuating pressure p against the mean flow U over the area L^2, that is, $\rho_w u U^2 L^2$. Corresponding to the two integrands of (4.2), we find the radiation efficiency as

$$\eta \sim \frac{1}{6\pi} M^5 \left(\frac{\Delta}{l}\right)^2 \left(\frac{c_w}{c_m}\right)^4 M^2, \tag{4.4}$$

or

$$\eta \sim \frac{1}{6\pi} M^5 \left(\frac{\Delta}{l}\right)^2 \left(\frac{c_w}{c_m}\right)^4 \left(\frac{\rho_a c_a}{\rho_w c_w}\right)^2. \tag{4.5}$$

These two can be compared with the radiation efficiency of Q, the sound that would be generated by a pure, non-bubbly turbulent flow. This sound can be found by either calculating the leading term of Q or applying the Lighthill (1952) acoustic analogy to the corresponding geometry. It turns out that, if we symbolically still denote by Q the radiation efficiency of the pure turbulence sources, then

$$Q \sim \frac{1}{6\pi} M^5 \left(\frac{\Delta}{l}\right)^2.$$

It is now very clear that the effect of compressible motions of the bubbles on sound radiation is to modify the inherent turbulence sound by either $(c_w/c_m)^4 M^2$ or $(c_w/c_m)^4 (\rho_a c_a/\rho_w c_w)^2$, neither of which can exceed $O(1)$ for the ocean-sound problem.

Crighton & Ffowcs Williams (1969) have investigated sound production by a bubbly flow embedded in an infinite region of water. They conclude that bubble motions may induce monopole sources whose power output dramatically overwhelms the usual Lighthill quadrupole sound by $(c_w/c_m)^4$. In view of this our conclusion that the influence of bubbles in the ocean surface layer is essentially negligible seems to be a startling result. The contrast is entirely due to the presence of the ocean surface. It can be illustrated by examining their basic equation (2.3), which reveals that, apart from the usual turbulence quadrupoles, bubble motions associated with compressibility induce a distribution of monopoles. However, as was shown in their paper, these monopole sources are actually equivalent to isotropic quadrupoles, though their strength is bigger than that of the Lighthill quadrupoles so that the power output is amplified by $(c_w/c_m)^4$, and hence the radiation efficiency is of the order $(c_w/c_m)^4 M^5$. In the case of sound generation by surface waves, all the sources are near a density interface that is almost pressure-release. It is well known that sound from isotropic quadrupoles adjacent to such an interface will be either reduced by $(\rho_a c_a/\rho_w c_w)^2$ or degraded to an octopole field, that is, reduced by M^2. This is precisely what (4.4) and (4.5) predict. Since the two reduction factors usually offset the increased value due to the multiplying factor $(c_w/c_m)^4$, the bubble-induced monopoles are not more efficient than those inherent turbulence quadrupoles which, though also close to the ocean surface, *still* radiate a sound of efficiency of the order M^5 because the density interface cannot change the order of sound from quadrupoles with one, and only one, axis perpendicular to the mean position of the interface. Therefore when Crighton & Ffowcs Williams' equation (2.3) is solved in the boundary problem where the sources are located near the ocean surface, the dominant sound is still given by the usual Lighthill turbulence sources, and this again leads to the conclusion that the effect of the compressible motions of the bubbles can be completely ignored.

The fact that bubbles near the ocean surface are very weak acoustic sources has been experimentally observed (I. Roebuck 1987, private communication); the noise of bubbles produced by a ship propeller is mainly generated by those bubbles near the propeller, where the interactions between the bubbly flow and the solid propeller blades are very strong, while in the wake, which may stretch far downstream of the ship and last a long time after the passage of the ship, bubbles can hardly radiate any noticeable noise, owing to the fatal destructive interference with their negative images from the ocean surface.

Now we turn to the sound caused by whitecaps. When splashing water sprays fall down on to the water surface, they usually form patches of highly bubbly mixture on the surface, which are surrounded by relatively clean water. Thus these patches of whitecaps cause a momentum flux in the source region, which generates sound in the same way as that in which splashing water sprays radiate sound. However, unsteady whitecaps can be shown to be much less efficient as acoustic sources than water sprays. This is done in what follows.

For water sprays, we scale the mean density within the clouds of water drops as ρ_w and neglect the influence from the surrounding airflow that has a much smaller mean density ρ_a. The sound pressure, from (2.12), is then proportional to

$$\int_{V_0}\left[\rho\frac{\mathrm{D}u_3}{\mathrm{D}\tau}\right]\mathrm{d}^3y = \rho_w\int_{\text{sprays}}\left[\frac{\mathrm{D}u_3}{\mathrm{D}\tau}\right]\mathrm{d}^3y. \qquad (4.6)$$

In the case of whitecaps, the high concentration of air bubbles within the whitecaps makes the mean density much lower than ρ_w, so that we have

$$\int_{V_0}\left[\rho\frac{\mathrm{D}u_3}{\mathrm{D}\tau}\right]\mathrm{d}^3y = \rho_w\int_{V_0}\left[\frac{\mathrm{D}u_3}{\mathrm{D}\tau}\right]\mathrm{d}^3y-\rho_w\int_{\text{whitecaps}}\left[\frac{\mathrm{D}u_3}{\mathrm{D}\tau}\right]\mathrm{d}^3y. \qquad (4.7)$$

It can be recognized immediately that the first term on the right-hand side is negligible, because the integral is over a fixed volume bounded by linear surfaces (as shown in §3). Since each one of the whitecaps is produced by a falling cloud of water drops, the sprays and whitecaps can reasonably be assumed to have a similar structure in their time–space distribution. Hence the integrations in (4.6) and (4.7) over the source region give a comparable effect to both terms and the relative magnitude of the two is entirely determined by the integrands, that is, by the vertical acceleration of water sprays and whitecaps.

The vertical acceleration of water sprays, detached from the main water body, can be estimated as g, the gravitational acceleration. Turbulence-pressure fluctuation in the airflow may also contribute to $\mathrm{D}u_3/\mathrm{D}\tau$, but it is of the order $\rho_a uU/l\rho_w$, l now denoting the linear dimension of the water sprays. If we let c be the phase speed of surface waves and use the result from Phillips (1985) that $l \sim 0.24c^2/g$, the turbulence-induced acceleration can be shown to be of the order $0.09(u_*/c)^2 g$, u_* being the friction velocity and assumed to be of the same order as u. This is very much smaller than g, because u_*/c is much less than one for most energetic large-scale waves that are breaking (for the dominant wave in an active field being of the order 0.05).

The vertical acceleration of whitecaps can be estimated from the surface wave field. As shown in the Appendix, the dominant motion of the ocean surface arises from gravity waves so that whitecaps, attached on the water surface, are all convected by gravity waves. Hence, with σ denoting the angular frequency of the dominant wave, we can scale $\mathrm{D}u_3/\mathrm{D}\tau$ as $\sigma^2\zeta$ for whitecaps. Here ζ can be calculated by integrating its frequency spectrum $\Phi(\omega) = \alpha g u_*/\omega^4$ (Phillips 1985), where α is a constant in the range from 0.06 to 0.11. From this we find that $\zeta \sim (\alpha g u_*/3\sigma^3)^{\frac{1}{2}}$. On scaling σ as g/U, it follows that the ratio of (4.7) to (4.6) is $(\alpha u_*/3U)^{\frac{1}{2}}$. The comparison

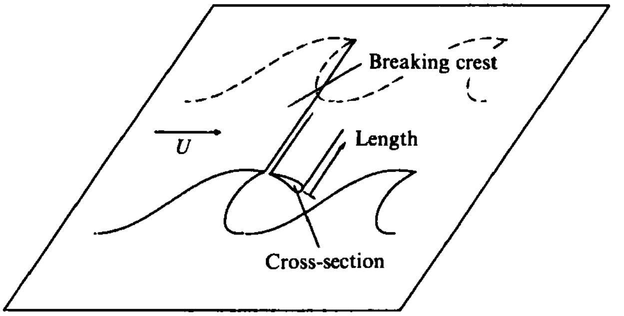

FIGURE 3. The schematic definitions of the cross-section and length of a water spray.

can also be made in terms of acoustic power output W, which can be easily calculated from the pressure field (2.12). It can be deduced that

$$W_{\text{whitecaps}} = \frac{\alpha u_*}{3U} W_{\text{sprays}}.$$

Considering that $\alpha = 0.06 \sim 0.11$ and $u_*/U \approx 0.05$, we see that the power ratio of the spray-generated sound to the whitecap-generated sound is at least of the order 10^3; the latter can be neglected.

5. A model for the calculation of sound power

From all the arguments in the preceding sections, we can conclude that the splashing water sprays are the major generator of underwater sound. Now we present a model to estimate the sound power from this mechanism. We establish the relation among the radiated sound power, the wind and the surface wave field. From (2.12) the sound pressure is

$$p(\boldsymbol{x}, t) = \frac{-\cos\theta T(\theta)}{2\pi c_{\mathrm{w}}|\boldsymbol{x}|}\frac{\partial}{\partial t}\int_{V_0}\left[\rho_0\frac{\mathrm{D}u_3}{\mathrm{D}\tau}\right]\mathrm{d}^3\boldsymbol{y}, \tag{5.1}$$

where
$$\int_{V_0}\left[\rho_0\frac{\mathrm{D}u_3}{\mathrm{D}\tau}\right]\mathrm{d}^3\boldsymbol{y} = \rho_{\mathrm{w}} g V(t), \tag{5.2}$$

and $V(t)$ denotes the total volume occupied by water sprays above the main surface. The production of water sprays by breaking waves is an unsteady process so that $V(t)$ is a function of time. Approximately, the instantaneous spray-occupied volume is equal to the space-averaged cross-section $A(t)$ of sprays times their total length, which is defined as the dimension of the sprays along the wave crests (see figure 3). Letting $\xi(y_\alpha, t)$ represent the length of water sprays in unit surface area, we can then write $V(t)$ as

$$V(t) = A(t)\int_{y_\alpha}\xi(y_\alpha, t)\,\mathrm{d}^2y_\alpha. \tag{5.3}$$

On substituting (5.3) and (5.2) into (5.1) we find that

$$p(\boldsymbol{x}, t) = \frac{-\cos\theta g\rho_{\mathrm{w}} T(\theta)}{2\pi c_{\mathrm{w}}|\boldsymbol{x}|}\int_{y_\alpha}\frac{\partial}{\partial t}[A(t)\,\xi(y_\alpha, t)]\,\mathrm{d}^2y_\alpha. \tag{5.4}$$

The acoustic power can be estimated from this by following Lighthill's (1952) idea of 'incoherent small eddies'. In doing so we take the ensemble average of the square of the pressure (5.4). One of the two y_α-integrals in the squared pressure can be carried out explicitly by assuming that the sources are well correlated only with those sources that are located within a surface area of small dimension, characterized by the coherence length scale l. Then, integrating the result over a hemisphere far from the sources and dividing it by the total area of the source region, it can be shown that W, the sound power radiated from unit area of source region, is

$$W = \frac{g^2 \rho_w}{6\pi c_w^3} \int_{r_\alpha} \overline{\frac{\partial}{\partial t}[A\xi(y_\alpha, t)] \frac{\partial}{\partial t}[A\xi(z_\alpha, t)]} \, \mathrm{d}^2 r_\alpha \tag{5.5}$$

with $r_\alpha = z_\alpha - y_\alpha$ and the overbar denoting the ensemble average.

Here the quantity $\partial A\xi/\partial t$ is essentially the variation of the spray-occupied volume. It is mainly determined by those surface waves that are breaking. The lack of our knowledge of these highly sporadic and sparse breaking waves (their space–time distribution, say) causes difficulty in estimating the sound. Nevertheless, Phillips (1985) suggested that in dealing with statistical characteristics of breaking waves it is relatively satisfactory to use the phase speed c as a measure of the scale of the breaking events. We adopt Phillips' scheme to estimate the magnitude of $\partial A\xi/\partial t$. This procedure starts with defining a distribution, $B(c)$ say, in the wave-speed space such that $B(c)\,\mathrm{d}c$ represents the contribution to $\partial A\xi/\partial t$ from those waves that have velocities in the range c to $c+\mathrm{d}c$. The total contribution can then be found by integrating $B(c)\,\mathrm{d}c$ over all possible velocities. This is similar to the idea of 'linear summation', which seems to take no account of the nonlinear interactions among waves of different velocities. It is legitimate, however, for our purpose. Our concern here is the properties of the sprays from breaking waves, not the nonlinear waves themselves. In a fully nonlinear surface wave field, the wavelength and frequency of a breaking wave cannot be specified in any unambiguous way, but its phase velocity is a well-defined quantity. Hence any water sprays in the source region can always be related to the definite velocity of the wave that produced them. Provided we ignore the back influence of the sprays on their origin, the main surface wave field, the aggregate effects on the spray production of the whole wave field can then be accounted for by the integration of the contribution from all the individual waves. This is the viewpoint that regards the fully nonlinear motions in the main wave field as independent of the sprays that have already been produced and are related to the wave field only through the phase velocity of the waves which created them.

In this view, for a spray-generating surface wave of speed c, we have from Phillips (1985) that the distribution of length per unit area of breaking front per velocity element is $\xi = b_1 u_*^3 g/c^6$, where b_1 is a constant of the order 0.01. And at this velocity we also have $A = b_2 c^4/g^2$ and $\partial/\partial t = b_3 g/c$ with $b_2 \approx 0.06$ and b_3 of order one. Thus it can be deduced that

$$B(c)\,\mathrm{d}c = \left(\frac{b_1 b_2 b_3 u_*^3}{c^3}\right) \mathrm{d}c,$$

from which it is found that

$$\overline{\left(\frac{\partial}{\partial t}[A\xi(y_\alpha, t)] \frac{\partial}{\partial t}[A\xi(z_\alpha, t)]\right)^2} = \frac{4 b_2^2 b_3^2 c_0^4 n^2}{g^2}, \tag{5.6}$$

where c_0 is the minimum phase speed of surface waves capable of producing water sprays and n is the expected number of breaking crests passing a given point in unit time, which is given by Phillips (1985) as

$$n = \frac{b_1 u_*^3 g}{4 c_0^4}.$$

This result can be used to reveal the frequency character of the spray-generated noise. If we assume that each one of the breaking crests produces a water spray containing approximately 10^2 water drops, the number of water drops passing a fixed point in unit time is roughly $n \times 10^2$. This is a typical frequency of the source process and also the frequency character of the sound generated by this source. The quantity n can be estimated by regarding c_0 as being of the same order as but somewhat bigger than the minimum phase speed of waves in the saturation range, which is given by $(4gT/\rho_w)^{\frac{1}{4}}$, T being the surface tension (Phillips 1977). Surface waves of phase speed lower than this value are decaying; they cannot break. Hence the typical frequency of the sound radiated can be estimated as

$$\frac{10^2 \times b_1 \rho_w}{16T} \left(\frac{u_*}{U}\right)^3 U^3.$$

This is basically a function of the uniform wind speed U; for U varying from its threshold value for breaking, of about 5 m/s, to the extreme high wind of 30 m/s, the typical frequency ranges from 20 Hz to 3750 Hz. For instance, in a storm of typical wind speed of 15 m/s, the peak frequency is approximately 500 Hz. Due to complex situations in the real ocean, it is impossible to distinguish the spray-induced sound from the actual measurements in any unambiguous way, so that the predicted frequency character cannot be checked quantitatively. Nevertheless, some observations and analyses do support this prediction (e.g. Wenz 1962); spray-generated noise in the ocean is found very likely to be peaked at these predicted frequencies.

On substituting (5.6) into (5.5) and scaling $d^2 r_\alpha$ as l^2 we find

$$W = \frac{2b_2^2 b_3^2 \rho_w c_0^4}{3\pi c_w^3} l^2 n^2.$$

From the definitions of l and n it is apparent that their product is the velocity with which the water sprays move (the distance travelled by a spray in unit time being equal to both the velocity and ln). The velocity of water sprays can be scaled on the wind speed because sprays can intuitively be regarded as being blown away from surface waves by wind. On this account we write $ln = b_4 U$, b_4 being of order one, and the sound power output becomes

$$W = \mu \frac{2}{3\pi} \frac{\rho_w c_0^4 U^2}{c_w^3}, \tag{5.7}$$

where we have combined all proportionality coefficients as a single constant μ, that is $\mu = b_2^2 b_3^2 b_4^2$. Taking account of the definitions of all these coefficients we may specify μ in the range $0.01 \leqslant \mu \leqslant 1.0$.

The result (5.7) is our theoretical prediction of the sound generated by water sprays. It shows a quadratic dependence of the overall sound power on wind speed and relates the sound to the surface wave field through the minimum phase speed of waves capable of producing sprays. Since sound in the natural ocean is usually measured, and reported in the literature, in terms of the power spectrum $P(f)$, the integration of which with respect to frequency f yields the mean square pressure, it is necessary to develop a model to convert the measured spectra into sound power in order to compare our theory with the measured data.

We choose to work with an 'infinitely deep ocean' model. This assumption is apparently reasonable since the wavelength of the sound concerned is very much smaller than the ocean depth where the measurements are made, and the reflection of the sea bottom in this situation is very poor. On this account, the sound field far

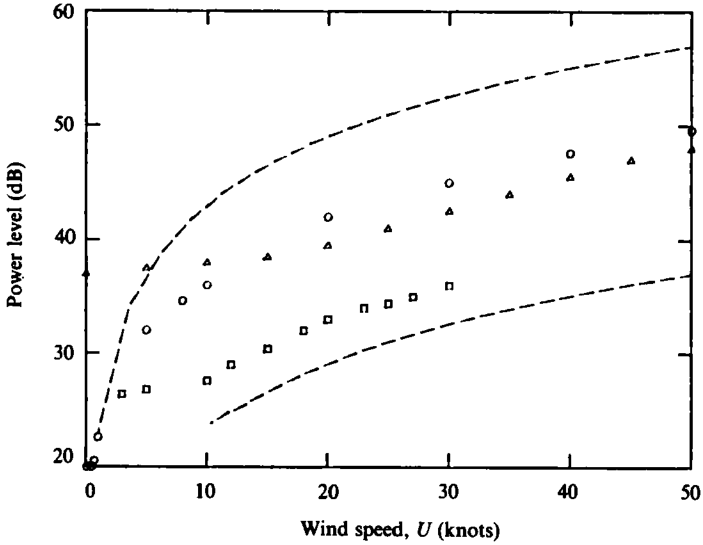

FIGURE 4. Sound power per unit frequency from unit area of source region. The dashed curves are the maximum and minimum values of the theory (5.7) and the measurements are respectively taken from Perrone (1969), Morris (1978) and Knudsen *et al.* (1948).

away from the source region is all outgoing, and the energy flux in the radial direction can be written as

$$I_r = \frac{\overline{p^2}}{\rho_w c_w} = \frac{3\cos^2\theta}{2\pi|\boldsymbol{x}|^2}\Pi \tag{5.8}$$

where $\cos\theta = x_3/|\boldsymbol{x}|$ and Π is the total sound power. The last step of this relation can be verified by integrating it over a hemisphere of radius $|\boldsymbol{x}|$. Now we define W' as the sound power radiated from unit area of source region. The sound power radiated by sources in the surface area δs is then given by $\delta\Pi = W'\delta s$. On the other hand, we have from (5.8)

$$\delta I_r = \delta\left(\frac{\overline{p^2}}{\rho_w c_w}\right) = \frac{3\cos^2\theta}{2\pi|\boldsymbol{x}|^2}\delta\Pi,$$

which then leads to

$$\delta\left(\frac{\overline{p^2}}{\rho_w c_w}\right) = \frac{3\cos^2\theta}{2\pi|\boldsymbol{x}|^2}W'\delta s. \tag{5.9}$$

This is the relation between the radial energy flux in the far field and the source element δs. If we further assume that the sources are energetically unrelated, namely that the contributions to I_r from disjoint surface elements are incoherent, we can replace the variational symbol δ by the differential symbol d and integrate this relation over the whole source region to derive the total contribution from all surface elements. Denoting by h the depth of the observation point and using the polar coordinates (r, ϕ) in the horizontal plane, we can write $\cos\theta = -h/|\boldsymbol{x}|$ and $|\boldsymbol{x}|^2 = r^2+h^2$. For a large patch of random source distribution of dimension L, the sound power per unit surface area W' can be supposed to be uniform over the source region $r < L$ and vanishing outside it. Hence we have by integrating (5.9)

$$\frac{\overline{p^2}}{\rho_w c_w} = \frac{3W'}{2\pi}\int_0^{2\pi}\int_0^L \frac{h^2}{(r^2+h^2)^2}r\,\mathrm{d}r\,\mathrm{d}\phi = \tfrac{3}{2}W'\frac{L^2}{L^2+h^2}. \tag{5.10}$$

By expressing the mean square pressure in terms of the power spectrum $P(f)$, the sound power from unit area of source region is found to be

$$W' = \frac{2}{3\rho_w c_w}\left[1+\left(\frac{h}{L}\right)^2\right]\int_f P(f)\,df. \tag{5.11}$$

Since the dimension L of the source region is usually much bigger than the depth of the observation point, the second term in the bracket can be neglected, and (5.11) simplifies to

$$W' = \frac{2}{3\rho_w c_w}\int_f P(f)\,df. \tag{5.12}$$

We compare our theoretical prediction (5.7) with this equation. The results are shown in figure 4, where the overall sound power level (sound energy per unit area of source region per unit time in decibels) is depicted as a function of the wind speed. Considering the somewhat arbitrary choice of the constant μ in (5.7) within the specified range, we illustrate our theory by plotting the maximum and minimum values of the predictions (the dashed curves). The comparison is made with three sets of experimental data, two of which, from Perrone (1969) and Morris (1978), respectively indicated by triangles and squares, are calculated from (5.12) according to the originally reported spectral data. The other set (circles) is taken from Knudsen *et al.* (1948), who gave the measurements in terms of the mean square pressure so that (5.10) is used in this case to evaluate W'. Apparently the measurements can all be fitted by choosing suitable values of μ. This indicates that the overall sound power in the ocean can very satisfactorily be described by the quadratic dependence theory, though the pressure spectra themselves have been observed to vary with wind speed as $U^{1.5}$ to $U^{3.6}$ at different frequencies. Figure 4 also shows that the predictions are too poor at low wind speed (below 10 knots). The sound is then dominantly generated by the turbulent airflow; there are no breaking waves.

6. Conclusions

Sound generation by breaking waves on the ocean surface has been examined. The splashing water sprays from the breaking have been identified as the main cause of the underwater noise. It has been shown that unsteady momentum fluctuation caused by water sprays generates a dipole sound field in the same way as that analysed by Ffowcs Williams (1986) for sound production by mixed turbulent flow in aeroacoustics. Radiation from air bubbles in the surface layer has also been examined, and it has been found that their effect on sound generation can be completely ignored in any event. Bubble motions associated with compressibility in this case can only radiate a sound that is essentially of the same order as the inherent turbulence sound, while the momentum defects arising from the replacement of water elements by the air bubbles are of much less importance than that resulting from the water sprays. Sound power output due to splashing water sprays has been evaluated in a model of the real ocean, which agrees well with observations from the natural ocean.

The author would like to thank his supervisor, Professor J. E. Ffowcs Williams, for his continuous encouragement and guidance. The Studentship from the University of Cambridge (Chancellor's Fund) is also acknowledged.

Appendix

A model problem is solved to determine the dominant motion of the ocean surface beneath which there exists a bubbly mixture layer of thickness ϵ, characterized by the mean density ρ_m and sound speed c_m and bounded above by the air of ρ_a and c_a and below by the water of ρ_w and c_w. We examine the response of the three-layered fluid system to a harmonic point excitation of strength q located in the bubbly layer at $(0,0,b)$ of a coordinate system (x,y,z), b being positive. Letting $\omega > 0$ be the angular frequency of the source and suppressing the time dependence $\exp(-\mathrm{i}\omega t)$, the governing equation in terms of the pressure perturbation p can be written as

$$\nabla^2 p + \frac{\omega^2}{c^2} p = q\delta(x, y, z-b). \tag{A 1}$$

When this equation is used in different layers, an appropriate subscript should be added; for example, the equation for the pure water region $z \leqslant 0$ should be with reference to p_w and c_w. The conditions connecting the solutions in different layers are

$$p_a = p_m, \quad \frac{1}{\rho_a}\frac{\partial p_a}{\partial z} = \frac{1}{\rho_w}\frac{\partial p_m}{\partial z} \quad \text{on } z = \epsilon \tag{A 2}$$

and

$$p_m = p_w, \quad \frac{1}{\rho_m}\frac{\partial p_m}{\partial z} = \frac{1}{\rho_w}\frac{\partial p_w}{\partial z} \quad \text{on } z = 0, \tag{A 3}$$

which simply state that the pressures and normal velocities on both sides of an interface between any two layers are equal.

The solution to the set of equations (A 1)–(A 3) can be obtained by the method of Fourier transformation. The results, representing the pressure field induced by the source, can be used to calculate the velocity field, and hence the displacement at the surface $z = \epsilon$, through the momentum equations. In this way we derive ζ as

$$\zeta(x,y) = \frac{q}{4\pi^2\omega^2}\int_\infty [\rho_a \gamma_m \cos[\gamma_m(\epsilon-b)] - \mathrm{i}\rho_m \gamma_a \sin[\gamma_m(\epsilon-b)]] \times \frac{\gamma_m}{F_d(k_\alpha)} \mathrm{e}^{-\mathrm{i}(k_1 x + k_2 y)}\,\mathrm{d}^2 k_\alpha, \tag{A 4}$$

where

$$F_d(k_\alpha) = \rho_m \gamma_m(\rho_a \gamma_w + \rho_w \gamma_a)\cos(\gamma_m \epsilon) - \mathrm{i}(\rho_a \rho_w \gamma_m^2 + \rho_m^2 \gamma_a \gamma_w)\sin(\gamma_m \epsilon), \tag{A 5}$$

and $\gamma^2 = \omega^2/c^2 - k_\alpha^2$ with appropriate subscripts and the choice of branches such that when real, $\gamma > 0$ and when purely imaginary $\mathrm{Im}(\gamma) > 0$. This ensures that waves are all outgoing and finite at infinity. It is a straightforward manner to evaluate the k_α-integral in (A 4) through the use of the residue theorem, the result being

$$\zeta(x,y) = \frac{\mathrm{i}q}{2\omega^2}\sum \frac{\gamma_m k[\rho_a \gamma_m \cos[\gamma_m(\epsilon-b)] - \mathrm{i}\rho_m \gamma_a \sin[\gamma_m(\epsilon-b)]]}{\partial F_d(k)/\partial k} H_0^{(1)}(kr),$$

where $r^2 = x^2 + y^2$ and the summation is over all possible zeros of $F_d(k)$, k being the modulus of k_α. This solution can be investigated numerically. It can also be analysed asymptotically by letting

$$\frac{\rho_a c_a}{\rho_w c_w} \to 0, \quad \frac{c_m^2}{c_w^2} \to 0, \quad \rho_m \to \rho_w.$$

It can be deduced that

$$\zeta(x,y) = \frac{qc_{\mathrm{m}}}{2\mathrm{i}\omega^3\rho_{\mathrm{w}}} \sum \frac{\gamma_{\mathrm{m}}(k^2-\omega^2/c_{\mathrm{w}}^2)\sin[\gamma_{\mathrm{m}}(\epsilon-b)]}{1+\epsilon(k^2-\omega^2/c_{\mathrm{w}}^2)^{\frac{1}{2}}} H_0^{(1)}(kr), \tag{A 6}$$

where $\gamma_{\mathrm{m}}^2 = \omega^2/c_{\mathrm{m}}^2-k^2$ and k is simply determined by

$$\sin(\gamma_{\mathrm{m}}\,\epsilon) = \left(\frac{\omega\epsilon}{c_{\mathrm{m}}}\right)^{-1}(\gamma_{\mathrm{m}}\,\epsilon). \tag{A 7}$$

Now, if we include gravity g in the equations (A 1)–(A 3), the function $F_{\mathrm{d}}(k)$ can be shown to also have a zero near $k = \omega^2/g$. This gives rise to surface gravity waves, with

$$\zeta(x,y) = \frac{\mathrm{i}q\omega^2}{g^2\rho_{\mathrm{w}}} \exp\left[-\frac{\omega^2}{g}(\epsilon-b)\right] H_0^{(1)}\left(\frac{\omega^2}{g}r\right). \tag{A 8}$$

A direct comparison can be made between (A 8) and (A 6) to decide the dominant contribution to ζ. To this end we choose to work with a typical term in (A 6). This is reasonable because the summation is actually over finite terms; the sum of finite terms cannot fundamentally increase the order of the individual terms. In fact, (A 6) contains at most three terms for the ocean-sound problem. It is apparent that (A 6) has more than three zeros only if $\omega\epsilon/c_{\mathrm{m}}$ exceeds $\frac{13}{2}\pi$. For sound of frequency 100 Hz, this means that the thickness of the bubbly layer must exceed 1.3 m, a situation that rarely happens in the natural ocean. Furthermore, since the depth of the source location $\epsilon-b$ is assumed much smaller than the typical wavelength in the ocean, the exponential factor in (A 8) is effectively unity and $\sin[\gamma_{\mathrm{m}}(\epsilon-b)]$ in (A 6) can be replaced by $\gamma_{\mathrm{m}}(\epsilon-b)$. On this account, and approximating γ_{m} and k by their maximum values, we find the ratio of (A 6) to (A 8) as

$$\frac{\omega(\epsilon-b)}{c_{\mathrm{m}}}\,\frac{1}{1+\epsilon\omega/c_{\mathrm{m}}}\left(\frac{g}{\omega c_{\mathrm{m}}}\right)^{\frac{3}{2}}.$$

This is a vanishingly small quantity, since the three factors in it are all very much smaller than one. Hence we have shown that the main contribution to ζ comes from gravity waves.

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